

BEYOND HEISENBERG'S UNCERTAINTY LIMITS

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1. INTRODUCTION

Heisenberg's uncertainty relations state that it is impossible to predict the result of a measurement with a precision lying beyond its limits. In practice this formula defines, in standard quantum mechanics, the limits of the previsual measurement space accessible to any measurement. Beyond it is impossible to perform any concrete measurement.

In the last years a strong effort was made to develop causal quantum physics in the spirit of de Broglie and Einstein. These efforts were carried out mostly with the help of a recent mathematical tool, the wavelet analysis, developed precisely to overcome some inconveniences of the Fourier non-local analysis. As a consequence of this work, a more general expression for the uncertainty relations, containing the usual ones as a particular case, were derived. These more general uncertainty relations define a predictive measurement error space including the usual Heisenberg space. In this new space the prevision of the precision of a result of a measurement depends only of the basic measuring tool used in a concrete experiment. If one wishes to predict the result of a measurement with greater precision it is necessary to change the basic measuring tool. In the language of local wavelet analysis this corresponds to say that one has to use a mother wavelet of a smaller scale. By changing the scale of the basic wavelet it is possible, in principle, to scan all the predictive measurement space.

For some time those ideas had no experimental support, even if the new approach explained the experimental evidence, which is not a surprise, since the new relations contain the usual ones as a particular case. Now, thanks to the recent development of a new generation of microscopes that have a resolving power that goes far beyond Abbe's limit of half wave length, there are measurements, done everyday, that are outside the Heisenberg's space.

2. WAVELET ANALYSIS AND THE NEW UNCERTAINTY RELATIONS

The wavelet local analysis [1] was developed in the early eighties by Jean Morlet, a French geophysicist working in oil prospection. He devised this mathematical tool for

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denoising the very sharp seismic signals. These early attempts were developed and formalized by Grossmann, Meyer, and many others till the wavelet local analysis was made into a powerful mathematical tool, competing side by side with the Fourier non-local analysis. Today, there is a whole universe of scientific literature dealing with the all-different aspects of wavelets. This field is developing so fast that even the very definition of wavelet that at the beginning seemed fairly settled, is today an uncertain ground that has lead some to authors to say that the precise definition of wavelet is a kind of scholastic question. Here, only the basic property of finitude of the wavelet is retained.

2.1. WAVELET LOCAL ANALYSIS VERSUS NON-LOCAL FOURIER ANALYSIS

In order to understand the conceptual and practical importance and grasp the deep significance of this new local analysis by wavelets it will be useful to make a short comparison with the non-local Fourier analysis.

The Fourier analysis must be considered non-local or global because its basic elements, its constituting bricks, are monochromatic harmonic plane waves, infinite both in space and time.

The non-local or global character of the Fourier analysis can be easily understood with the following example.

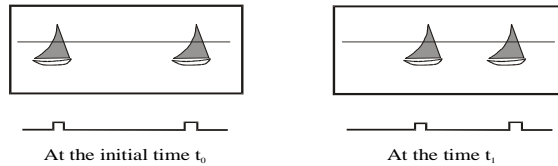


Figure 3.1 – Digitalized images to Fourier analyze

Suppose that it is necessary to record, line by line, the first image shown in Figure 3.1, for instance, in a CD. In order to simplify the problem, let us consider only the line indicated in the picture. The plot of the intensity of the picture along that line is also represented below in the same picture. In order to Fourier represent this variation in intensity it is necessary to find the amplitudes, frequencies and phases of the infinite monochromatic harmonic plane waves that sum up reproducing the initial function. This immensity of plane waves interferes negatively in all points of space except in those two regions where the interference is constructive.

Let us now suppose that, for instance, the first ship moves to the right, while the other remains in the same position. This situation is represented in the second image of Figure 3.1. Next, we ask how this motion of the first ship is to be Fourier represented. In this type of analysis, both the first and the second ship are made of the same infinite harmonic plane waves. In these circumstances, a modification in the position of the first ship implies naturally a modification in the amplitude frequency and phase of all the waves, so that their interference is constructive now only in the new position. This means that, even though the two ships may seem separate entities, they are in fact, treated as a whole entity in this process of non-local Fourier analysis and reconstruction.

Hence, a modification in the position of one of the ships implies a simultaneous modification of the constituents of the other, no matter how distant they are from one another. This mathematical consequence of dealing with infinite harmonic plane waves, determines the very deep roots of the non-locality of the usual quantum mechanics. The non-local or global character of Fourier analysis has also serious technical drawbacks, namely when, as is the case, one wants to register video images. In general, as is very well known, from video image to video image there are only slight variations. Nevertheless, in this type of non-local or global analysis, a minor modification on one region of the image implies the necessity of analyzing and rebuilding the whole image.

If, instead of the non-local Fourier analysis, one uses local wavelet analysis, the modification of the position of one ship as nothing to do with the other, if the two objects are sufficiently far one from another. In this case, the group of wavelets that sums up to recreate one object is completely independent from the wavelets corresponding to the reconstruction of the other ship. This is precisely the reason why this type of analysis is called local analysis by finite waves.

The difference between the two types of analysis is shown graphically in Figure 3.2.

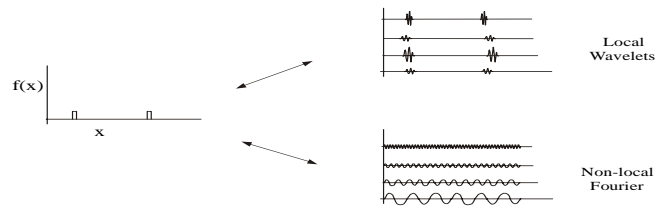


Figure 3.2 – Comparison between local wavelet analysis and Fourier non-local analysis

Another important advantage of the local wavelet analysis is related to the fact that, in order to represent a given signal, one is free to choose different types of basic wavelets. In these circumstances, a wise choice of the basic wavelet can further increase the rate of compression and the quality of the reconstruction, or the denoising capability.

2.2. THE NEW UNCERTAINTY RELATIONS

Analytically, the usual uncertainty relations are a mathematical consequence of the non-local Fourier analysis [2]; if, instead of infinite harmonic plane waves, one uses the local wavelet analysis to represent a quantum particle, the form of uncertainty relations may consequently change [3].

In order to make the derivation of the new uncertainty relations more comprehensible, it is convenient to do it step-by-step, in parallel and with the usual derivation. This process is shown in the following sketch, Figure 3.3

Derivation of the uncertainty relations

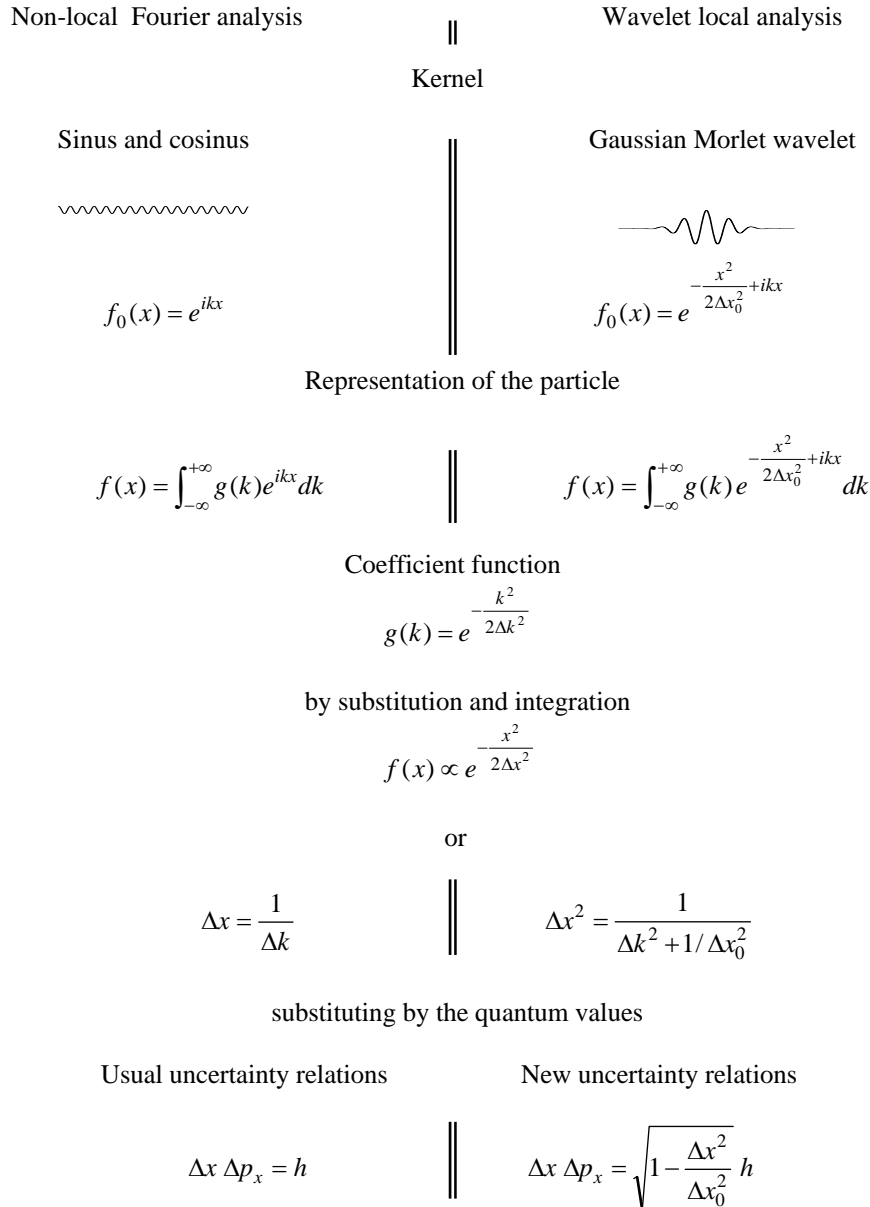


Figure 3.3 – Derivation of the uncertainty relations.

From the table, Figure 3.3, it is seen that the new uncertainty relations derived with the local wavelet analysis have the form

$$\Delta x^2 = \frac{h^2}{\Delta p^2 + h^2 / \Delta x_0^2} . \quad (2.1)$$

The same analysis could, in principle, be done with other mother wavelets. The Gaussian Morlet wavelet [4] was chosen among the other possibilities because of its interesting properties:

- From the mathematical point of view, it has a very simple form. Therefore, the necessary calculations can fully be carried out without approximations.
- Another very interesting property of this wavelet comes from the fact that, when its size increases indefinitely, it transforms itself in the kernel of the Fourier transform. In this sense, this local analysis contains the non-local Fourier analysis as a particular case. When the size of the basic wavelet Δx_0 is sufficient large, the new relation transforms itself into the old usual Heisenberg relations, which is a very satisfactory result
- It allows a reasonable representation of fair localized particle with a well-defined velocity.
- Written in terms of space and time, it is a solution of a Schrödinger non-linear master equation [5].

The plot of the uncertainty is shown in Figure 3.4. The new uncertainty relations are represented by the solid line for three finite values of the size of the basic wavelet. The usual Heisenberg uncertainty relations, which correspond to the case $\Delta x_0 \rightarrow \infty$, are shown as a dashed line.

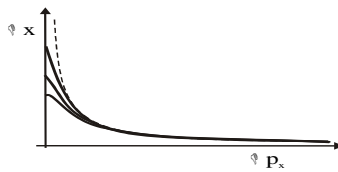


Figure 3.4 - Solid line: new uncertainty relation for three different finite values of size the basic wavelet. The dashed line represents the usual Heisenberg uncertainty relations.

2.3. GENERALIZED MEASUREMENT SPACE

The new, more general uncertainty relations were derived in a causal framework, assuming that the physical properties of a quantum system are observer independent, and, even more, that they exist even before the measurement. Naturally due to the unavoidable physical interaction during the measurement process, when the other conjugated observable is to be measured, the quantum system does not remain in the

same state. In last the instance, the precision of the measurement, for a non-prepared system, depends on the relative size between the measurement basic apparatus and the system upon which the measurement is being done.

Until now, the most basic interacting quantum device is the photon. Nevertheless if the photon has an inner structure, as is assumed in the Broglie model, it will imply the ability to perform measurements beyond the photon limit. Since in the derivation of the new uncertainty relations, the quantum systems were assumed to be described by local finite wavelets, the measurement space resulting from those general relations must depend on the size of the basic wavelet used. As the width of the analyzing wavelet changes, the measurement scale also changes. This can be seen in the following plot Figure 3.4.

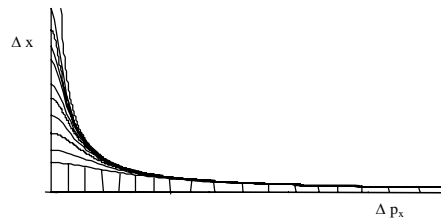


Figure 3.4 – Wavelet measurement space

From this picture it is seen that, as the width of the basic wavelet Δx_0 changes, all the measurement space is scanned. This space is only limited by Heisenberg's space. The smaller Δx_0 , the greater is the precision of the position measurement. That is: the smaller is the uncertainty Δx , for any value of the error in the momentum. Given that the new relation contains the usual as a particular case, it implies that the measurement space available to the general uncertainty relations is the whole space, as shown in Figure 3.5.

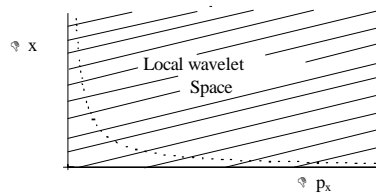


Figure 3.5 – Measurement space available to the general uncertainty relation

These results are rather satisfactory, because in this causal paradigm the quantum measurement process depends, in last instance, on a standard used. We are, in principle, free to choose the size of the mother wavelet Δx_0 more suitable for the measurement precision we want to attain.

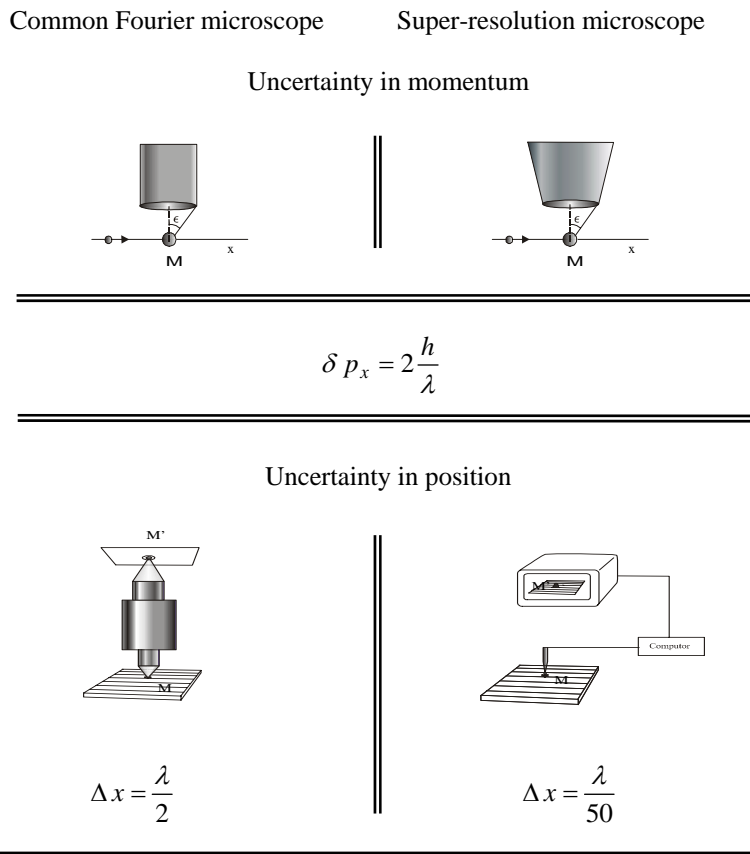
3. BEYOND HEISENBERG'S MEASUREMENT SPACE

The middle of the eighties saw the development of super-resolution microscopes. The first of this new type of microscopes was developed for electrons by Bennig and Rohrer [6], who received the Nobel Prize for the discovery. The principal characteristic of these microscopes is that their resolution goes far beyond those given by Abbe's rule of half wavelength. Abbe derived his theoretical resolution rule based on the Rayleigh diffraction criteria for the separation of two circles in a far field.

Super-resolution microscopes were also developed in 1984 by Pohl et al. [7] for the optical domain. Initially, these microscopes had a spatial resolution of $1/20$, ten years later [8] they attained resolutions of $1/50$ or even better.

There are many types of super-resolution optical microscopes, as can be seen in pertinent literature. In most straightforward of these microscopes the light emitted by the sample is simply collected by the sensing probe.

In order to see if there are some very special experimental situations where the predicted errors of the two conjugated observables do not lie in Heisenberg's measurement space, it is convenient to consider the well-known Heisenberg microscope in parallel with the super-resolution microscope optical microscope.



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Product of the two uncertainties

$$\Delta x \Delta p_x = h \quad \parallel \quad \Delta x \Delta p_x = \frac{1}{25} h$$

From the table, one sees that there is a very significant difference of 1/25 in the products of the uncertainties related to the measurement of the two conjugated observables, position and momentum. For the common Fourier microscope, this product lies well within Heisenberg's measurement error space $\Delta x \Delta p_x \geq h$. For the super-resolution microscope, we have $\Delta x \Delta p_x = h/25$, that lies outside $\Delta x \Delta p_x < h$ Heisenberg's measurement space, and is therefore contained in the more general wavelet precision measurement space.

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4. REFERENCES

1. A. Grossmann and J. Morlet, *SIAM J. Math. Anal.* **3** (1989),723; C.K Chui, *An Introduction to Wavelets*, Academic Press, N.Y. 1992.
2. N. Bohr, (1928) – *Como Lectures, Collected Works*, Vol. 6, North-Holand, Amsterdam, 1985.
3. J.R. Croca, *Apeiron*, **4** (1997), 41.
4. P. Kumar, *Rev. Geophysics*, **35** (1997), 385.
5. Ph. Gueret and J.P. Vigier, *Lett. Nuovo Cimento*, **38** (1983), 125; J.P. Vigier, *Phys. Lett. A*, **135** (1989), 99.
6. G. Benning and H. Roher, *Reviews of Modern Physics*, **59** (1987), 615.
7. D.W. Pohl, W. Denk and M. Lanz, *Appl. Phys. Lett.* **44** (1984), 651.
8. H. Heiselmann, D.W. Pohl, *Appl. Phys. A*, **59** (1994), 89.