# Classifying the computational power of stochastic physical oracles

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Consider an algorithm requesting information from an external source – an *oracle* – the terminology originates with Alan Turing [7]. Emil Post [6] used oracles to study computability.

However, suppose the external source is not a pure mathematical entity but a *physical device or environment*. Suppose the requests are for measurements of physical quantities. We call this external source a *physical oracle*. Algorithms with physical oracles may be found in measurement experiments, and in controlling machines. We ask: *What is the computational power of adding a physical oracle? How does the computational theory depend upon the physical theories and models?*

In [2,3] we developed a computability and complexity theory for physical oracles. The computational classification needed non-uniform complexity classes [1], especially $P/\log\star$ and $BPP//\log\star$ [5]. Using case studies, we formulated axioms expressing properties common to wide classes of physical systems [4].

Here we review physical oracles and report new results broadening their scope by using *non-deterministic physical systems*. Physical oracles with probabilistic theories we call *stochastic physical oracles*. We examine examples of three types of non-deterministic systems, those that that are physically nondeterministic, as in quantum phenomena; physically deterministic but whose physical theory is non-deterministic, as in statistical mechanics; and physically deterministic but whose computational theory is non-deterministic caused by error margins. We prove:

**Theorem 1.** *Let $SPO$ be the axioms for stochastic physical oracles. Let $P$ be a physical system whose behaviour depends upon a physical quantity or parameter $\sigma$. Suppose $P$ satisfies the axioms of $SPO$. Then: a set $A \subset \{0,1\}^\star$ is decidable in polynomial time by a Turing machine with physical oracle $P$ and unknown parameter $\sigma$ if, and only if, $A \in BPP//\log\star$.*

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